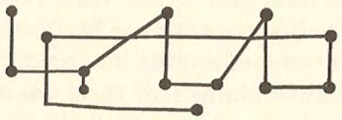
THE WILSON INSTALLATIONS



WARREN BURT

> . . . Do I have to tell you about the spiritual cannibalism of the culture, our culture, which has been bombarding us with ultrasensory overstimulation aiming to reprocess us into fulltime consumption machines, stealing above all from us our time (not an inch of time without an imprint of message), and even our very sense of time (to be measured in lengths of no more than one message unit each) under the guise of entertainment, and even of 'art', commoditizing the eternal, hyping the primal? Our time is the sine qua non of our identity. We need to take extreme measures to reclaim it for ourselves and each other.
>
> — Benjamin Boretz, *Interface*, Part V.

THE WILSON INSTALLATIONS are a set of three very large-scale live solo electronic music performance installations that deal with extended tuning systems, the relation of tuning to timbre and spatiality of

sound, concentration of attention for extended time periods, site specific installation and performance, live interaction with algorithmic generation of melody and harmonic choices, and the notion of the artist re-gaining control over the presentation and production of their own works. The works were written between 2000 and 2004, and the initial impetus for each one came from certain music theory articles by Ervin M. Wilson, musicologist and music scale theorist of Los Angeles. The works are *The MOSsy Slopes of Mt. Meru: The Meru Expansion* (2000–2004) for two laptop computers and two synthesizers; *Pythagoras' Babylonian Bathtub* (2003) for three laptop computers and three synthesizers; and *Saturday in the Triakontahedron with Leonhard* (2000–2004) for one laptop and one software synthesizer. Each work lasts a minimum of one hour, and preferably much longer, although shorter versions have been performed. All three pieces have been performed: *The MOSsy Slopes* in 2003 at the Rechabite Hall, Northcote, Melbourne; *Pythagoras* in 2003 at Cecil St Studio in Fitzroy, Melbourne, and at the 2003 Sonic Connections Festival at the University of Wollongong; and *Saturday* in a shortened form at the Sonic Connections Festival at the University of Wollongong in 2004. All three pieces also exist in recorded form, with the entire cycle available (from http://www.warrenburt.com) as a four CD set. I'm still waiting for the right circumstances to emerge where I can put on all three pieces in the ideal way I envisage, as a day long, or several days long, series of installation/performances, where over the course of a day, all three works would be performed.

The large-scale nature of the works came about for a number of reasons. Firstly, each work originated in researches into certain tuning systems proposed by Ervin Wilson. Each of these systems:

1) the Meru Prastara, also known as Pascal's Triangle;

2) the Scale Tree of Charles Peirce;

3) and the Euler-Fokker Genus 3 5 7 9 11 13;

has the ability to generate whole families of interrelated scales and harmonies. In my developments of them, in fact, they were even more fertile than Wilson proposed, each one leading to between 60 and 167 interrelated scales. With this large body of resources, it seemed to me that a great length of time was necessary in order to explore the potentials of these scales. But beyond that, I also had clear political reasons for the extreme length of the pieces. As indicated in the quote from Benjamin Boretz that prefaces this text, I, too, felt that extreme measures were necessary to reclaim a sense of time and identity. I had done long pieces in the past—for example, *Le Grand Ni*, from 1978, was a seventy minute long composition that explored a specific just intonation scale played through specially made loudspeakers—metal signs with transducers bolted to them, hung throughout an exhibition space. Harmonically, these new pieces are different from my earlier long pieces. In *Le Grand Ni*, for example, I explored one scale for 70 minutes. In each of these new pieces, I have a huge harmonic vocabulary—hundreds of scales—to explore. Even working at the rate of harmonic change of traditional commercial music, it would, as stated earlier, still take hours to explore all of those scales in even a superficial way. And given the "bullet-point" consumerist treatment of time in contemporary society, I felt that extreme measures were necessary to reclaim a sense of time and concentration on sound that was, I felt, being lost.

In the late 1990s, I had not done long pieces—I had become an enthusiastic participant in the improvised music scene. I liked the continual ongoing sense of "practice" that happened in the improv scene—the fact that you were always developing ideas and almost immediately trying them out in performance. But as I believe that the nature of the music should dictate the nature of the performance space and manner of presentation, and not vice-versa, I found that these harmonic ideas of mine just didn't fit into the variety show concept of 10–20 minute "sets" that had developed in the Melbourne improvisation scene. Also, since I wanted people to listen mainly to the harmonies of these pieces—and I here acknowledge that mostly, when people hear music, they're not concentrating on the harmonies, or even on pitch—gesture, timbre, rhythm, and social context all seem more important in what we might uneasily term "average" listening—I felt I needed a different venue for this music than the smoke-clogged, acoustically bad, primarily-designed-for-socializing venues that the Melbourne improv scene used. I also found, however, that in my quest for a convivial, small-scale music making (which was one reason why I went into the improv scene in the first place, and which has been a quest of mine since the 1970s, and which, incidentally, has also been a concern of Ben's over the years), I was also out of sync with much of contemporary computer music presentation, which favoured large scale presentations on institutionally installed multi-loudspeaker setups. ("The Spectacle and Computer Music" (Burt, 1999) has more detail on this).

I've always felt that artists should be in control of the means of production, performance and dissemination of their work. In terms of technology, this has led me to the ideal of personally owning all the equipment necessary for the performance of the piece at hand. And if economic conditions meant that large equipment was not affordable, ways of working with cheaper equipment to produce the desired result should be found. My own version of Shumacher's *Small is Beautiful* was expressed as, "You should be able to carry all the gear you need to the gig on public transport." With these pieces, we've expanded slightly, but not by much (you need a rental car).

I also acknowledge a contradiction here—I want to do long, subtle pieces which require very controlled conditions for their performance and reception—but I want to do them in a small-scale, low-budget convivial way. For example, for the Melbourne presentation of these pieces, I used the Rechabite Hall, a small venue, available at the time for about 50 dollars a day, and a dance rehearsal space, Cecil St. Studio, and not, to pick two grand venues at random out of many, the BMW Edge Atrium in Federation Square, or Story Hall at Royal Melbourne Institute of Technology. Using these smaller dance and music spaces meant that I could afford to present the work myself, when I wanted to, and not have to go through bureaucracy and external organizations. It also meant that the home-scale sound equipment I own could be used in performance, without the necessity of hiring unwieldy large-scale sound systems. I'm a great fan of using smaller sound systems to explore the acoustics of each individual space. In *Le Grand Ni*, the speakers were nothing more than scrap metal and transducers which shaped the sound in interesting ways. In terms of sound systems, although it looked grand, it was actually as low-tech as the cassette recorders we were also using back then. For these pieces, normal loudspeakers were required, but I felt the use of smaller systems gave me more control and focus with the sound.

These pieces also have a unique form—what I call a performance installation. That is, the works are very long, and live in a particular space for a long time period—preferably hours, if not days. However, I interact with the computers, improvisationally making decisions about the shaping of the sound, so they're also performances. They have to have flexibility to be done, as I said, as shorter live performances, and also as sculptural installations of CDs played through a system of several loudspeakers, but the form of presentation I prefer is that of the performance/installation, where concentration is directed to the sound itself, where the audience is free to enter and leave at will, and where people can change their position as they will.

In this article, I want to deal mostly with how I derived the harmonies of the pieces, and how I perform them. But, as mentioned, the pieces result from a combination of four interests of mine: tuning systems, live electronic performance, the use of processes and number structures for generating music, and small-scale convivial performance and presentation. This led me to question each of these interests. Specifically:

Why did I get into tuning?

Why did I get into live electronic music performance?

Why did I get into process (or algorithmic) composition?

Why did I evolve the self-reliant (or small-scale) performance ethic?

Long answers to each of these could be given, but as succinctly as possible: I got into tuning because I couldn't do it very well, and there were inspiring people around who were doing it very well indeed (Harry Partch, for example), so I decided to see if I could do it too. Also, the mathematics involved looked absolutely intimidating. Mathematics was something I wanted to learn more about, but I'd never given sufficient attention. I found that I could do both the mathematics and the tuning (and the retraining of my listening habits), and it became fascinating for me to think that pitch and interval quality could be such a deep and endlessly expanding field. An epiphanic moment for me was hearing the Indian composer and singer Pandit Pran Nath sing a descending semitone glide. For about two minutes. He told us what he was going to do, and then gave us a demonstration. I felt the floor fall away from under me. For those two minutes, I was literally floating in one of the biggest harmonic spaces in the universe, as he showed us, with absolute vocal control, that there was a lot of territory to explore there, if only one knew how to concentrate on it. Thirty years later, I'm still fascinated by the many different shades of sound that can exist within what we think of as a single interval.

I got into live electronic music performance because it was something I could do myself, without needing to rely on other performers, and because, when I first learned electronics—in the late 1960s—there were just so many possibilities available with electronics that acoustic instruments couldn't do, that it almost instantly became my native compositional language. Since then, I've been involved with the development of many electronic music tools. I was involved in the development of the Serge analog synthesizer, built my own electronic music machines, such as the Aardvarks series (Aardvarks IV—digital random control voltage generation and waveform assembly machine, 1972–75; Aardvarks VII, just intonation counters and dividers, 1977–78; and Aardvarks IX, microcomputer based control and sound-synthesis system, 1981–85), and most recently, have been involved in the development of software tools, such as John Dunn's SoftStep and ArtWonk, Martin Fay's Vaz Modular, and Ross Bencina's AudioMulch.

Algorithmic composition was first shown to me by Joel Chadabe in the late '60s. The combining of control voltages on the early synthesizers to make musical patterns that were otherwise unavailable was an impetus to further investigation, as was contact with composers who were using processes to generate their music, such as John Cage, Cornelius Cardew, Salvatore Martirano, and Chadabe himself. Later, my interests in numerical systems, such as chaos of various kinds, was provoked by my inability to "do the math." This was a challenge, so I learned the mathematics necessary for each new project/interest in turn. The possibilities of these kinds of composing thrilled me. I could get a larger variety of musics than if I just "felt" my way through traditionally expressive kinds of composing. For me, variety of output is something I value highly. As well, the idea of interacting with a "would be intelligent" or "semi-intelligent" machine system is something I like. I enjoy creating systems which give me information that I can respond to at the moment of performance. In the early 1920s, Ernst Krenek had a similar experience when he first encountered counterpoint: "I was fascinated by the notion that music was not just a vague symbolisation of *Gefühl* instinctively conjured up into pleasant sounding matter, but a precisely planned reflection of an autonomous system of streams of energy materialized in carefully controlled tonal patterns." (Krenek, 1971)

Finally, the idea of the self-reliant composer was first suggested to me by Kenneth Gaburo, who pointed out that I should own my own equipment, as it was unlikely that I would have access to institutional facilities forever. In 1972, when he pointed this out, owning one's own was financially more difficult than today, but still possible. This interest was then reinforced by my collaborations with Ron Robboy, in the groups Fatty Acid and YCMA, and later with our collaborative feature-length Super-8 film, *Der Yiddisher Cowboy*. In each of these projects, the self-reliant, low-tech ethic was applied to a variety of parameters, competence being among them. (If as composers, we could structure pitch and duration, we surely could also structure degrees of both physical and conceptual competence). Later, when I moved to Melbourne, work with composers such as Ron Nagorcka and Ernie Althoff on the development of a low-budget ethic refined and developed this idea, with a slightly more political edge than it had had in California.

Thirty years later, I find myself still exploring these issues, and I still find myself fascinated by the worlds of serious inquiry that remain, and that keep opening themselves up for exploration.

Each of the three pieces is performed on laptop computers, using software and hardware synthesizers. The computers are set up on a table in the middle of the room, allowing the audience to wander around, and hopefully even look over my shoulder, and sound is heard through a small portable sound system. Generally, there have been two larger and two smaller loudspeakers. The aim of these has been to spread the sound evenly throughout the space, so that the effect of the space itself on the sound can be heard clearly. There is not much panning or directing of sound to any particular position in space, except minimally in *Saturday*, the third piece. I wanted the feel of the sound in space to actually be a product of the interaction of my sounds and the space itself. Each family of scales was generated with a particular mathematical diagram or phenomenon. In each piece, a different process, involving some elements of randomness, was used to choose what pitches were playing and in which combinations. Some people may call this a new Pythagoreanism, in that it deals heavily with number and the concept of music being number made audible. There may be an element of truth to this, but I feel that because I'm doing all this to obtain and explore certain qualities of sound, I'm not as involved with the mystical nature of number as the historical Pythagoreans were. (Although recently, I've been reading the moral philosophy of some of the later Pythagoreans, such as Iamblichus, Porphyry, and especially Plotinus, and I've been liking what I'm reading very much). For me, in these piece, the idea of serious listening to very subtly different shades of sound is paramount, and it shaped my choice of structures and how and where I choose to present them.

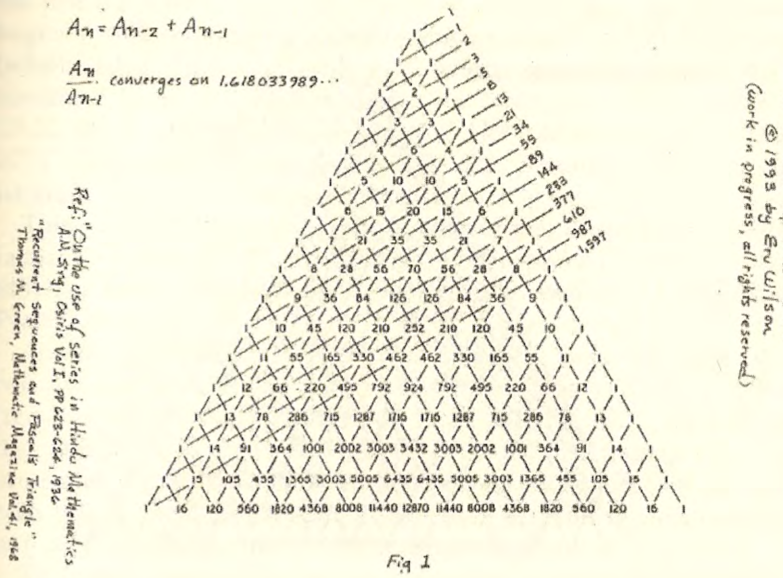
I would like to deal with each of the pieces now in turn, showing a little bit about how their harmonies were evolved, and how I perform and compose with them.

I. *The MOSsy Slopes of Mt. Meru.*

This piece uses sine-waves playing scales made from additive series found in the diagram known as the Meru Prastara, or, in the West, Pascal's triangle. (The approach taken here was to use the numbers of the series to generate frequency relationships to generate scales. A completely different approach, applying the relationships of the triangle to the piano in twelve-tone tuning, was taken by Tom Johnson in *Music for 88* (Johnson, 1988). In Wilson's 1993 article, *The Scales of Mt. Meru* (Wilson, 1993), following up on work by Thomas M. Green

(Green, 1968) and A. N. Singh (Singh, 1936), he shows how an infinite series of recurrent sequences can be derived by taking the sums of different diagonals of this diagram. It was first described by the Hindu mathematician Pingala as early as 200 BCE, and other triangular number diagrams existed in Hindu mathematics as early as 2000 BCE, according to Ernest McClain (McClain, 1976). Wilson has suggested that the recurrent sequences found in the Meru Prastara could be useful as a source for musical scales. Example 1 shows the triangle with the first of the recurrent number series derived from its diagonals.

In the triangle, each number of each lower row consists of the sum of the two numbers diagonally above it. Then, if parallel diagonal lines are drawn across the diagram (in this case, parallel with the line from the third number down on the left outer edge to the second number down on the right outer edge), and the numbers that those parallel lines intersect are added up, the result is the additive sequence shown on the right side of the diagram. The series in this diagram happens to be the well known Fibonacci series, in which each subsequent number



EXAMPLE 1

is the sum of the two numbers preceding it. Each different set of diagonal lines yields a different additive sequence, with different rules. For example, in Example 2, the diagonals parallel the line from the fourth number down on the left outer edge to the second number down on the right outer edge. This additive series (the sums of the numbers the parallel diagonal lines intersect) is different than the Fibonacci series. In this series, each new number is the sum of the number preceding it and the number three elements before it in the series. As stated above, each new series has a different derivation rule, but each new series is an additive series made by adding some combination of elements that preceded it. In each diagram, there is also a statement like "A_n/A_{n-1} converges on . . . 1.618033989." This refers to the ratio between each two adjacent elements of the series. For series A in Fig. 1, the ratio 8/5 = 1.6, while the ratio 144/89 = 1.6179775 . . ., the ratio 1597/987 = 1.6180344 . . ., etc. The farther out you go in each sequence, the closer each ratio gets to that

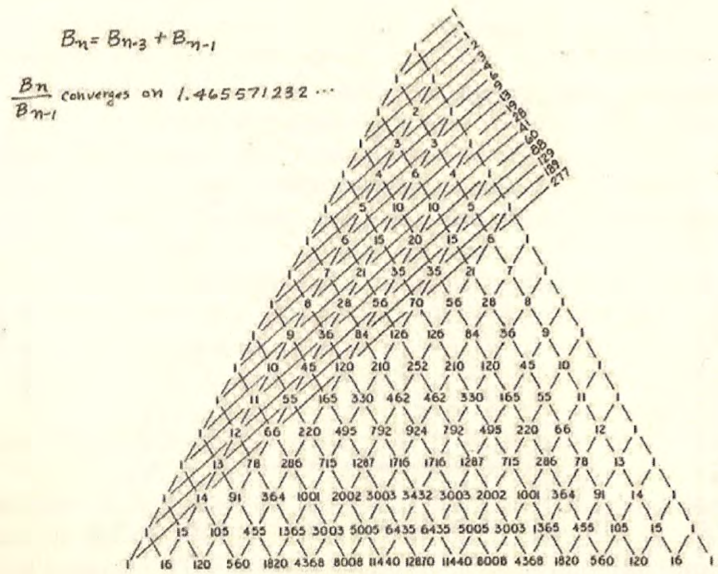


Fig. 2

EXAMPLE 2

"convergence number." In 2002, I explored the scales derived from the first five of these sequences. In 2003, I extended this to work with scales derived from the first 11 of the additive sequences.

This is all very interesting from a mathematical point of view (and I admit that I do find these mathematical explorations extremely pleasurable), but how can this be used in music, and more specifically, in tuning? In my 2002 paper "Developing and Composing with Scales Based on Recurrent Sequences" (Burt, 2002), I deal with the derivation of scales from these sequences in great detail. We here summarize the method of scale derivation more briefly. For the first sequence, for example, the Fibonacci series, it would be possible to take each element of the series as a harmonic of a fundamental and then reduce each harmonic to be within an octave. The set of notes that resulted would be the scale.

SCALE DERIVATION FROM NUMBER SERIES

Using the method given above, what scale results from, let us say, the first 23 unique Fibonacci numbers? (Because the Fibonacci series begins with 2 ones, and has a two (which reduces to one) and an eight (which reduces to one), the first 23 unique elements of the series are contained in the first 26 elements of the series, which are 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181 6765 10946 17711 28657 46368 75025 121393. The resulting scale is given in Example 3.

This scale has been used by a number of people, including Stephen James Taylor of Los Angeles. Following this procedure, we can derive different scales from each additive series contained within Pingala's, or Pascal's, triangle.

Another possibility presents itself, however. Since the ratio of any two adjacent members of this series eventually converges on a number, (remember "A_n/A_{n-1} converges on 1.618033989 . . ."), we could use that number as a ratio (taking itself over one and finding how many cents are in that interval), and then pile up those intervals on top of each other, assembling a scale in that manner. This is a generalization of the "Pythagorean" method of making scales.

The term Pythagorean is usually used to describe a scale made of piling up perfect fifth intervals (the ratio 3/2, or 702.0 cents) on top of each other, and then octave reducing them. For my purposes, I wish to use the term "Pythagorean" to refer to ANY scale made by taking a chain of N copies of ANY single interval, and reducing that to an

	Ratio	Cents	Fib. Number
0:	1/1	0.000	(1, 2, 8)
1:	4181/4096	35.559	(4181)
2:	17/16	104.955	(34)
3:	17711/16387	134.830	(17711)
4:	9/8	203.910	(144)
5:	75025/65536	234.101	(75025)
6:	305/256	303.199	(610)
7:	5/4	386.314	(5)
8:	323/256	402.468	(2584)
9:	21/16	470.781	(21)
10:	5473/4096	501.739	(10946)
11:	89/64	570.880	(89)
12:	1449/1024	601.010	(46368)
13:	377/256	670.105	(377)
14:	3/2	701.955	(3)
15:	1597/1024	769.378	(1597)
16:	13/8	840.528	(13)
17:	6765/4096	868.849	(6765)
18:	55/32	937.632	(55)
19:	28657/16384	967.920	(28657)
20:	233/128	1037.023	(233)
21:	121393/65536	1067.191	(121393)
22:	987/512	1136.288	(987)
23:	2/1	1200.000	(1, 2, 8)

EXAMPLE 3

octave. (In the freeware tuning software Scala (Op de Coul, 2010) this generalized form of scale generation is called a "linear temperament.") In the case of the convergence number of the Fibonacci series, 1.618033989 . . ., the ratio 1.618 . . ./1 is approximately 833.090297c. 1.618 . . . is the number Phi, the "Golden Section." (A recent essay showing applications of Phi to musical scales is Kak, 2004. Phi can also be expressed as a series of infinite continued fractions. McLaren, 1991 shows how to generate scales using this method in great detail.)

We could take a chain of these intervals, add them up and see what we get. Here is a chain of four of these intervals, for demonstration purposes (to fit these values within an octave, subtract either 1200 or 2400 cents. These altered values are given in parentheses where applicable):

0

833.0903

1666.1806 (1666.1806 – 1200 = 466.1806)

2499.2709 (2499.2709 – 2400 = 99.2709)

3332.3612 (3332.3612 – 2400 = 932.3612).

Put as a scale, this becomes (listed in cents):

0

99.2709

466.1806

833.0903

932.3612

1200.

This is an unusual, but not unpleasant pentatonic scale. In fact, if you extend this scale out to a moderate number of degrees, you'll find that it's almost identical to a scale made of the higher ratios of the Fibonacci series, without using the lower or first few elements of the series. For example, here is a scale made from stacking 23 of the 833.0903 cent intervals and reducing it to an octave. If the Fibonacci scale would be considered a "just intonation" realisation of the lowest elements of the Fibonacci sequence, this scale might be considered a "Pythagorean" realization of it (Example 4).

MOMENTS OF SYMMETRY

There is another question to consider when generating scales in this manner. That's the question of how many notes to include in the scale. Since these are infinite series, they could be extended forever. Is there any way to know "when to stop" in order to get usable scales that have

0: 1/1 – 0 cents
1: 30.174 cents
2: 99.271 cents
3: 129.445 cents
4: 198.542 cents
5: 228.716 cents
6: 297.813 cents
7: 327.987 cents
8: 397.084 cents
9: 466.181 cents
10: 496.354 cents
11: 565.451 cents
12: 595.625 cents
13: 664.722 cents
14: 694.896 cents
15: 763.993 cents
16: 833.090 cents
17: 863.264 cents
18: 932.361 cents
19: 962.535 cents
20: 1031.632 cents
21: 1061.806 cents
22: 1130.903 cents
23: 2/1

EXAMPLE 4

some sort of property of recognizability or coherence? I put this question to Erv Wilson, and he sent me a diagram of a metallophone he designed using the 23 note Fibonacci scale given above, grouped into interlocking sets of thirteen plus ten notes, in an analogous manner to the way the seven note white key and five note black key scales are interlocked on the piano keyboard. He told me that the property known as "Moments of Symmetry" might provide some way of grouping these new scales into usable subsets.

A "Moment of Symmetry" (MOS) occurs when, in piling up a chain of equal intervals, a scale with two and only two melodic scale degree sizes results. (In the world of mathematics, this is known as Myhill's property.) There have been a number of articles on the workings of Moments of Symmetry (Wilson, 1975a, 1975b, Chalmers, 1975), but briefly, if we were making a chain of 3/2 perfect 5ths (701.955 cents), we would generate MOS scales with 2, 3, 5, 7, 12, 17, etc degrees in them. All the rest of the scales (for example 4, 6, 8, 9 scale degrees) will have three kinds of melodic scale degree intervals in them. Ignoring the trivial examples of scales of two and three scale degrees, the lowest example in the chain of fifths scale where two MOS scales add up to a third MOS scale with NO intervening examples of a MOS scale is the sequence of five scale degrees plus seven scale degrees equal twelve scale degrees. For our Fibonacci scales, the lowest example of two scales that add up to a third with no intervening scales is ten scale degrees plus thirteen scale degrees equal twenty-three scale degrees.

For the Pythagorean version, the MOS property is exact. That is, a scale made of stacking the 833.0903c interval of convergence will exhibit MOS properties with scales of thirteen scale degrees and ten scale degrees. For the just intonation version, that is, the scale made with the numbers of the Fibonacci series itself, the MOS property, strictly speaking, does not exist. However, we can use the same scale degrees that make the MOS scale in the Pythagorean version to make an "approximate MOS" version from the Fibonacci series scale. The charm of using a scale made of only the mean "convergence interval" is that the scale is internally consistent. The charm of using the Fibonacci series (or any other number series) is that the just intervals at the bottom of the series are relatively farther away from the mean "convergence interval," giving more variety. You pays your money, and you takes your choice. Or, in the case of this piece (and this series of pieces in general), you can have it both (or more) ways, using several different ways of deriving scales from the aspects of the same number series.

In *The MOSsy Slopes of Mt. Meru (The Meru Expansion)*, I used the first eleven number series generated by Pingala's/Pascal's triangle. Each number series generated six related scales. For example, for the first, the Fibonacci scale,
($A_n = A_{n-2} + A_{n-1}$ (A_n/A_{n-1} converges on 1.618033989 . . .)
MOS: 13 + 10 = 23)
I made a 23 note just intonation scale using the numbers of the series as harmonics and reducing them to an octave. I then divided that scale up into an interlocked thirteen note and ten note subset. (The division is determined by the order of the numbers of the series. That is, the first thirteen numbers of the series generate the thirteen note subset, and the next ten elements of the series generate the ten note subset. When these are placed within an octave, they make interlocking

patterns.) The thirteen note subset and the ten note subset were also treated as separate scales, so that three scales were produced from the Fibonacci series—a twenty-three note scale, and two sub-scales of thirteen and ten degrees respectively, which interlocked.

I then made a twenty-three note Pythagorean scale, using the convergence interval (as a ratio of itself over one, and expressed in cents) as a generator interval (the interval that is used to make the chain of identical intervals). I then took the first thirteen elements of this Pythagorean scale, and used them as one subset, and the next ten elements of the scale were used as another subset. This gave me a total of six scales:

1) The first twenty-three elements of the number series as (octave reduced) harmonics. (Just scale)

2) The first thirteen elements of that series as a subset. (Just scale)

3) The next ten elements of that series as a subset. (Just scale)

4) An octave-reduced chain of twenty-three generator intervals. (Pythagorean scale)

5) The first thirteen elements of that series as a subset. (Pythagorean scale)

6) The next ten elements of that series as a subset. (Pythagorean scale).

With eleven number series, and six scales each, this should generate 66 scales. However, the convergence interval of series F is exactly the same as the convergence interval for series C, so there are not three unique Pythagorean scales for series F, although the just intonation scales are different. The rules for each of these eleven number series, the convergence number, and the lowest division into a MOS formation (where the third scale is the sum of the previous two, with no MOS formation intervening) are given in Example 5. Readers interested in seeing how each of these series is derived from different diagonals in the Pascal/Pingala diagram are referred to Wilson 1993.[1]

In performance of all these scales, a curious phenomenon occurs. Since the only timbre I use in this piece is a pure sine wave, and since many of the notes are quite long in duration, sustained chords are set up. If a listener moves their head, even slightly, the balance of the tones of the chords seems to change, sometimes quite dramatically. This is because the pure sine tones are setting up standing waves

$A_n = A_{n-2} + A_{n-1}$ (A_n/A_{n-1} converges on 1.618033989 . . .)
MOS: 13 + 10 = 23

$B_n = B_{n-3} + B_{n-1}$ (B_n/B_{n-1} converges on 1.465571232 . . .)
MOS: 11 + 9 = 20

$C_n = C_{n-3} + C_{n-2}$ (C_n/C_{n-1} converges on 1.324717957 . . .)
MOS: 7 + 5 = 12

$D_n = D_{n-4} + D_{n-1}$ (D_n/D_{n-1} converges on 1.380277569 . . .)
MOS: 15 + 13 = 28

$E_n = E_{n-4} + E_{n-3}$ (E_n/E_{n-1} converges on 1.220744085 . . .)
MOS: 10 + 7 = 17

$F_n = F_{n-5} + F_{n-1}$ (F_n/F_{n-1} converges on 1.324717957 . . .)
MOS: 7 + 5 = 12

$G_n = G_{n-5} + G_{n-2}$ (G_n/G_{n-1} converges on 1.236505703 . . .)
MOS: 13 + 10 = 23

$H_n = H_{n-5} + H_{n-3}$ (H_n/H_{n-1} converges on 1.193859111 . . .)
MOS: 7 + 4 = 11

$I_n = I_{n-5} + I_{n-4}$ (I_n/I_{n-1} converges on 1.167303978 . . .)
MOS: 13 + 9 = 22

$J_n = J_{n-6} + J_{n-1}$ (J_n/J_{n-1} converges on 1.28529903 . . .)
MOS: 14 + 11 = 25

$K_n = K_{n-6} + K_{n-5}$ (K_n/K_{n-1} converges on 1.13472413 . . .)
MOS: 6 + 5 = 11

EXAMPLE 5

within the room, and each position in the room then has a different set of amplitudes of each waveform. This means that at each point in the room, the balance between the tones of any given chord will be different. And this difference will happen in whatever room the sound is played in, and be different, based on the size and shape of the room interacting with the frequencies of the piece. So in this piece, the spatiality of the sound is a direct by-product of an interaction between the particular architecture the piece is played in, and the frequencies of the piece itself. Here, timbre, tuning, and spatialization are intimately linked.

The piece is played on two laptop computers, linked by MIDI, running John Dunn's Softstep algorithmic control program. The larger laptop is also running Martin Fay's Vaz Modular software synthesizer, while the smaller computer controls a Korg XD5R hardware synthesizer. Both synthesizers are programmed to produce the same sine wave timbres, the main difference being in envelope lengths and note durations.

Pitches and rhythms of the piece are chosen with an algorithmic process I devised. Since I was using additive sequences, formed by the addition of previous elements, I thought that the "accumulator" module in Softstep would be useful as an appropriate melody generator. The accumulator, at every new clock pulse, adds the number at the value input to the sum of the previous values. When it reaches a (user-setable) maximum value, it reverses direction, subtracting value inputs from the sum until it reaches the (user-setable) minimum value, at which point it begins adding the value input again. For the value input, I had an array of lower members of the number series which generated the scale being used at the time. In the case of the Fibonacci series, this array was 1 1 2 3 5 8 13 21. Selection of elements from this array was under control of a shift-register feedback random number generation algorithm derived from John Roy and Joel Chadabe's work on their early 1970s random information generator called *Daisy*. Other people who used this algorithm at the time were myself (Burt 1975), Salvatore Martirano (Chadabe 1997), Greg Schiemer (Schiemer 1990) and Carl Vine (Vine 1977). The shift-register feedback module selected one of the eight members of the Fibonacci series. This was then put into the accumulator. The output of the accumulator was put into a modulo-n divider. This rescaled the output of the divider in an interesting way, and the resulting number controlled pitch selection from the current scale. This meant that the melodies used would consist only of intervals (number of scale degree steps) determined by the number series that generated the current scale. For the Fibonacci scales, that meant that only intervals of 1 2 3 5 8 13 or 21 steps would occur. Given the up-down nature of the accumulator, and the non-linear scaling that the modulo-n divider produced, this meant that ascending and descending melodic lines form the gestural vocabulary of the piece. These lines would not always alternate ascending and descending evenly, though, due to the size of the modulo-n divider and the limits of the accumulator sometimes being "out-of-sync." For example, if the accumulator produces a series which goes up and down from zero to 127 and back, but the modulo-n divider were set to 30, this would produce a series of ascending lines, then a series of descending lines, as the accumulator values are scaled by the divider (see Example 6).

Mod. Out can be read as the melody line, given in scale degrees. In this case, it is controlling a scale of ten steps per octave over 3 octaves. Notice how the series does not evenly alternate going up and down,

Accum. Out:	0	21	42	43	48	61	66	68	76	97	118	123	127	105	104	99	96	88	85	72	69	56	35	30
Mod. Out:	0	21	12	13	18	1	6	8	16	7	28	3	7	15	14	9	6	28	25	12	9	26	5	0
Accum:	17	14	1	0	5	etc.																		
Mod:	17	14	1	0	5	etc.																		

EXAMPLE 6

but that the up/down gestural shape predominates. Notice also that when the accumulator hits its maximum (or minimum) limit, it simply stops there, producing the occasional interval not given by our array of possible interval sizes. (The accum. out. line going from 123 to 127 (max limit) is an interval of four scale degrees, not one of our given Fibonacci numbered intervals). This occasional "error" is fine by me. It makes things a bit less consistent, and a bit more interesting.

Rhythmic durations (time points) are chosen with a similar algorithm. Each new time-interval is selected by an independent occurrence of the same shift-register feedback algorithm, which randomly selects from an array of possible durations. The values in this array are also chosen from the generating number series. In the case of the Fibonacci series scales, these values were 5 8 13 21 34 55 and 89. Furthermore, a new time-interval value is chosen every five notes, which means that while there may be (and frequently are) sequences of ten or fifteen notes having the same time-interval, at least every five notes there is the potential for a rhythmic change. Interestingly, these are heard more as tempo changes than as rhythmic changes. Even a series of five regularly spaced notes is enough to establish a sense of "tempo" and rhythmic constancy. But these five attacks don't always all have the same duration. Within a series of eight possible equally spaced pulses, only five are chosen for each scale. This produces occasional notes that have twice the duration of the other notes in the sequence. In the course of the piece, all possible combinations of five pulses out of eight are used.

The combination of these melodic and rhythmic algorithms produce a monophonic line which has a pleasing combination of predictability and unpredictability. Since most of the notes chosen will have long sustains (longer than the time until the next note begins), they will hang over, creating chords. In live performance, I turn on and turn off this melody making machine, producing a larger scale sense of phrasing and breathing in the piece. Additionally, the second computer is taking individual notes off this sequence, and choosing to play these with extremely long durations, thus assembling a series of quasi-drones, which play contrapuntally against the faster melodies produced by the first computer.

In live performance then, the machine, under my programming, makes certain decisions as to pitch, duration, sustain, and interval choice. I control the overall phrasing, the thickness of texture, and choose which of the 63 available scales I will explore next. In some performances, I've gone through the scales in linear order (first the just scales, then the Pythagorean ones), while in others, I've alternated just and Pythagorean examples of the same scale-type, while in still others, I've wandered freely among all the scales. For shorter performances, I tend to wander randomly around the available scales, while in longer performances, where time is allowed for subtle listening, I've tended to use more linear orders, so that a sense of long term harmonic motion, or at least, long term change of harmonic color, is also used.

A NOTE ON TIMBRE

In all three pieces, I used fairly plain, stable timbres. I did this for a number of reasons. Since I was dealing mainly with harmonies, and sustained harmonies at that, I wanted to use timbres that focussed listening on harmonic quality. So varying timbres, such as the honking of black swans, or socially loaded timbres, such as electric guitar distortion, didn't seem to be appropriate here. For *The MOSsy Slopes*, I used plain sine waves, in order to hear the harmonies clearly, and also to produce psychoacoustic interactions with the acoustics of the space the piece was performed in. As mentioned, one could move one's head and hear different tones come in and out based on where one was in the space. For *Pythagoras*, I used slightly more complex waveforms with a few harmonics to bring out the complexities of the scales used. For *Saturday*, I used classic electronic music frequency modulation timbres (fm), in an extremely corrupt analogy to the way the scales were created. That is, the scales in "Saturday" are made by multiplying numbers which are treated as harmonics with each other, as explained later (in "III: *Saturday in the Triakontahedron with Leonhard*"). In a burst of enthusiasm, and incorrect logic, I viewed the modulating of waveforms in fm as another form of multiplication, and decided to use those timbres on the basis of that incorrect comparison. It was some kind of multiplying of things against each other, anyway. Also, the scales used in that piece seemed "angular" to me, and the spectrum of the fm sounds seemed to match that "angularity," in some intuitive sense. In each of these pieces, there is at least a conceptual link between the harmony and the timbre.[2]

II. *Pythagoras' Babylonian Bathtub*

Because of the way he publishes his work—as sets of diagrams, without very much verbal explanation—Wilson's work can frequently seem enigmatic. I believe this is his way of making sure that people who work with his materials are really dedicated and interested—they have to "do the math" or the hard work involved in realising the import of the diagram before any use can be made of them. In my case, this has sometimes meant that papers he has sent to me have been often first read and dealt with in a fairly superficial manner. Then they sit around in the back of my head for several years, until I finally have an insight into them, whereupon I go back and study the paper again, in greater detail, and finally understand, to some degree of depth, aspects of the ideas I hadn't understood before. This was definitely the case with his paper "The Golden Horagrams of the Scale Tree" (Wilson, 1997) which I first received from him in 1997. I made one small piece with it (*64 Golden Chords* (1997)), treating each of the 32 Horagrams as a template for a chord and its associated additive synthesis timbres, but it wasn't until I received a paper from David J Finnamore, "5- to 9-tone, octave-repeating scales from Wilson's Golden Horagrams of the Scale Tree" (Finnamore, 2001), that I truly understood the import of Wilson's Scale Tree work.

Wilson first considered that the so-called "Farey Series" of order *n* was identical to the Lambdoma of the same order (Wilson, 1996) (The Lambdoma, based on ancient Greek tuning diagrams, is explained below). The Farey Series of order *n* is the ascending series of irreducible fractions between 0 and 1 whose denominators do not exceed *n*; the numbers 0 and 1 are included in the forms 0/1 and 1/1. For example, the Farey series order 4 is:

$$\frac{0}{1}\ \frac{1}{4}\ \frac{1}{3}\ \frac{1}{2}\ \frac{2}{3}\ \frac{3}{4}\ \frac{1}{1}$$

For musical purposes, Wilson proposed that the Farey series be inverted and symmetrically extended. The term 1/0 at the end is included for completeness, even though no interval or pitch results from it. This would extend the series to the following:

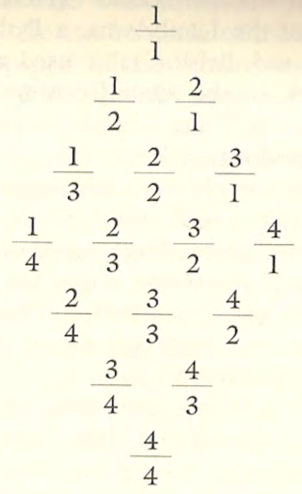
$$\frac{0}{1}\ \frac{1}{4}\ \frac{1}{3}\ \frac{1}{2}\ \frac{2}{3}\ \frac{3}{4}\ \frac{1}{1}\ \frac{4}{3}\ \frac{3}{2}\ \frac{2}{1}\ \frac{3}{1}\ \frac{4}{1}\ \frac{1}{0}$$

In "A Brief History of the Lambdoma" (Hero, 1995) Barbara Hero describes the history of the Lambdoma, a Pythagorean (and possibly earlier) multiplication and division table used as an anagram of ratios and musical harmonics. In the second century Nichomachus, among others, described it—an ordering of ratios of integers both above and below one. A basic lambdoma, of order four, is provided in Example 7. If the lambdoma from Example 7 is filled in, the lambdoma in example 8 is generated. You can see all the 1, 2, 3, 4 progressions in the numerators (left to right descending) and the denominators (right to left, descending). Then, when the ratios are reduced to within an octave—i.e. 3/3 = 1/1 and 4/2 = 2/1 etc, one can see that the ratio content of the reduced lambdoma is the same as Wilson's inverted and extended Farey series (Example 9).

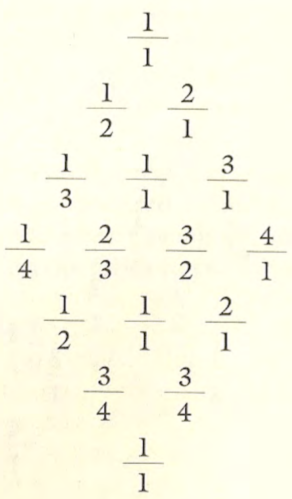
As for why Wilson felt that the series had to be inverted and extended, I think he took his cue from Partch. Partch insisted that every ratio be expressed both harmonically and subharmonically—that every ratio used in a scale should be inverted as well—so if you have 5/4 in a scale, you should also have 4/5 which within an octave is 8/5. His otonalities and utonalites are made of these intervals and their inversions, respectively—this means that each Partch scale has an odd number of tones, and is symmetrical. Partch said that he felt that any good just scale had this property of symmetry. So when Wilson takes

$$\frac{1}{1}$$
$$\frac{1}{2}\quad\frac{2}{1}$$
$$\frac{1}{3}\qquad\quad\frac{3}{1}$$
$$\frac{1}{4}\qquad\qquad\quad\frac{4}{1}$$

EXAMPLE 7



EXAMPLE 8



EXAMPLE 9

0 1 1 1 2 3 1
1 4 3 2 3 4 1

and inverts and extends to

0 1 1 1 2 3 1 4 3 2 3 4 1
1 4 3 2 3 4 1 3 2 1 1 1 0

he is making a Partchian otonality/utonality inversion for scale completeness.

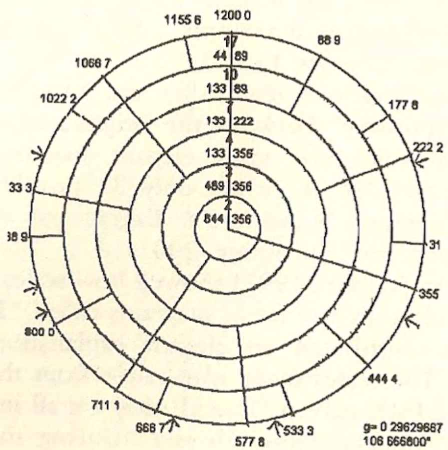
So the lambdoma and the Farey series of the same limit have all the same numbers. A lambdoma of order eleven, when octave reduced, is almost identical to Harry Partch and Augusto Novaro's idea of the tonality diamond (Partch, 1974; Novaro, 1927). My 1996 composition *Portrait of Erv Wilson* from *Harmonic Colour Fields* was based on the order eleven Lambdoma/Farey Series. An explanation of its musical usage can be found in Burt, 1997.

However, Wilson extended this conception further. He pointed out that when arrayed in a tree-like pattern, the ratios of successive Farey series made a series of paths based on the addition of their numerators and denominators. And this additive series of fractions would converge onto the Golden Section (that is, Phi) of the original two ratios. For two ratios a/c and b/d, the formula for finding the next element in the series is (a+b)/(c+d). For example, taking two starting ratios 1/3 and 2/7, this gives a series 1/3, 2/7, 3/10, 5/17, 8/27, 13/44, 21/71 etc. As the fractions get further and further along the series, they converge "Phi-way" between the original two fractions. Both Wilson and Finnamore have made elegant diagrams showing these paths. It turns out that there are only 32 possible additive paths through this diagram. Finnamore's diagram of this is especially beautiful and revelatory (Finnamore, 2001a).[3]

Further, Wilson (Wilson, 1997) showed how scales made with these ratios would develop in a series of diagrams called "Horagrams", and Finnamore has contributed an elegant explanation of how these diagrams work. The scales made with ratios from these diagrams are all, in my usage, Pythagorean. That is, they are all made by taking an additive series of identical intervals and reducing them to within an octave. In common with all Pythagorean-type scales, Moments of Symmetry occur, where a scale will have two and only two interval sizes. For example, taking a generator interval of 355.556 cents, and making a scale with it, we find that Moments of Symmetry occur with scales of two, three, four, seven, ten and seventeen scale degrees. Wilson used a diagram of concentric circles to show this. A circle of

1200 cents is used, and the generator interval line extends from the center. Each subsequent scale interval is then placed one ring of the circle outwards. If the scale generated is non-MOS, lines are added on that ring until the scale generated has MOS characteristics. In Example 10, made with Manuel Op de Coul's freeware "Scala" program (Op de Coul, 2010), the generator is expressed both in cents and as a number g, which shows the generator as a percentage of 1200 cents. Here, 1200 x g = 0.29629667 = 355.556 cents. The width of the angle in degrees (formed between the generator and the origin) is also given below g.

There are a number of ways to make scales from the ratios of the scale tree. The first is to take the ratio itself, treat the denominator D as the number of steps in an equal temperament, and the numerator N as an interval of N steps out of the D equal temperament to use as a scale generator. For example, take the ratio 8/27 given above. A generator of eight steps out of a 27 tone equal temperament yields an interval size of 355.556 cents. The horagram given in example 10 describes that scale, and shows that MOS scales can be found by taking (octave-reduced) scales of seven, ten and seventeen stacked intervals of that size. The second way of generating scales, and both Wilson and Finnamore have dealt with this in great detail in their papers, is to take



EXAMPLE 10

the convergence interval of each branch of this tree, and make a scale using that as the generator. For example, the series 1/3—2/7 stated above, eventually converges on 0.295685999. Multiplying this by 1200, we get an interval of 354.82 cents to use as our generator. Note that this is very close to the interval of 355.556 cents we derived from the 8/27 member of the series above. In fact, this is another example of the use of a convergent series that we saw in the Mt. Meru additive sequences. Both Wilson and Finnamore list a number of scales made with this second method, and Finnamore has written a number of compositions with scales of between five and nine notes made in this way. I realised that there was, however, a third way to generate scales from these ratios. That was to take the scale tree fractions as just intonation ratios, and to see what intervals they converged on. This was similar to the method used to derive the convergence intervals of the additive sequences of Mt. Meru. I took the convergent sequence 1/3 – 2/7 and treated each fraction as an octave-reduced just intonation ratio and this is what happened:

1/3 = 4/3 = 498c

2/7 = 8/7 = 231c

3/10 = 12/10 = 6/5 = 316c

5/17 = 20/17 = 281c

8/27 = 32/27 = 294c

13/44 = 52/44 = 289c

21/71 = 84/71 = 291c

etc. down to

377/1275 = 1508/1275 = 290.567018c.

When you use 290.567etc. as a Pythagorean generator, you get MOS scales at nine, thirteen, 21 and 33 degrees. Each of the 32 series of convergent fractions could be used to generate scales in this way. So for each of the 32 branches of the scale tree, there were three families of scales that could be generated. Some, based on N steps out of D-sized equal temperaments, and the convergence intervals (the Wilson-Finnamore method) might be very similar. Scales based on my method would usually be extremely different from both. I made a chart showing the possible Pythagorean scale generators available from the scale tree (Example 11). The Wilson Branch number and Finnamore

Wilson Branch no.	Finnamore Branch no.	Gen. N/M ET	Wilson/Finnamore Limit × 1200c Gen.	Burt Limit Ratio as Cents Generator
32	18	6/13	550.96c	1052.40c
31	10	9/20	541.38c	1022.02c
30	6	11/25	527.15c	975.90c
29	27	10/23	522.72c	961.29c
28	28	11/26	506.94c	908.21c
27	4	13/31	503.79c	897.416c
26	15	12/29	495.90c	870.12c
25	23	9/22	491.95c	856.245c
24	24	9/23	468.62c	772.148c
23	16	12/31	465.08c	759.03c
22	1	13/34	458.36c	733.82c
21	32	11/29	455.78c	724.025c
20	31	10/27	443.74c	677.68c
19	8	11/30	440.59c	664.38c
18	12	9/25	431.12c	627.746c
17	20	6/17	425.23c	603.937c
16	19	5/16	373.07c	377.38c
15	11	7/23	366.26c	345.47c
14	7	8/27	354.82c	290.56c
13	29	7/24	350.90c	271.295c
12	30	7/25	335.18c	191.96c
11	2	8/29	331.67c	173.75c
10	14	7/26	322.27c	123.96c
9	22	5/19	317.17c	96.35c
8	21	4/17	280.61c	1084.33c
7	13	5/22	273.85c	1042.11c
6	3	5/23	259.85c	951.27c
5	26	4/19	254.04c	912.13c
4	25	3/16	222.97c	686.24c
3	5	3/17	213.60c	611.92c
2	9	2/13	181.32c	328.32c
1	17	1/8	157.52c	84.70c

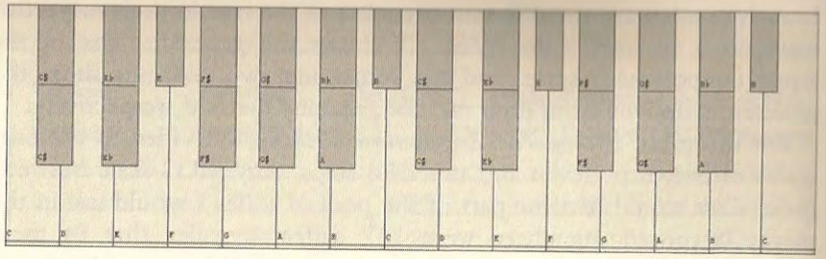
EXAMPLE 11

branch number are the different branches of the tree in their respective numbering systems. Gen N/M ET shows the generator size of the equal temperament scale, and the remaining two columns show the generators derived from their method, and my method, respectively.

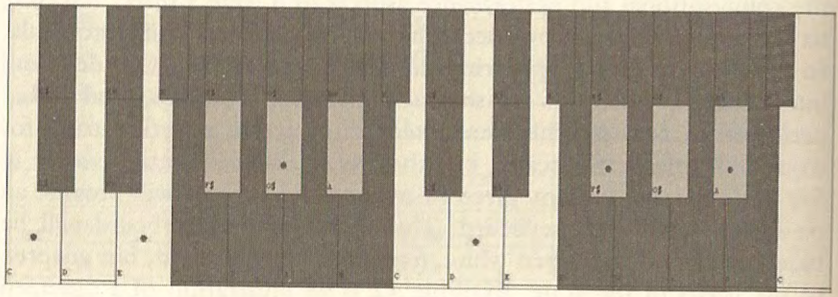
For this piece *Pythagoras' Babylonian Bathtub*, I decided to use only scales of between eleven and nineteen steps. Any MOS scale between those sizes would become part of the pool of scales I would use in the piece. It turned out there were 167 different scales that fit these criteria. It seemed to me that this was a suitably large set of harmonic resources to keep me interested for a while.

The harmonic world of *Pythagoras' Babylonian Bathtub* may have been inspired by the work of Ervin Wilson and David Finnamore, but the compositional and performance aspects of it were a direct response to the new performance devices Manuel Op de Coul built into Scala. In versions of Scala appearing in 2002 and 2003, Op de Coul introduced a series of on-screen keyboards, matrices, and other performance devices. This were intended to act as analytical tools for work with microtonal scales, but they were also eminently useable in live performance. For any given microtonal scale, Scala will provide an on-screen "split-key" keyboard. The design of this keyboard will be based on the normal seven white, five black key keyboard, but adapted to the needs of the scale. Example 12 is an illustration of a nineteen tone equal tempered keyboard. Note the split keys for the accidentals, and the "E♯" and "B♯" keys for nineteen tone.

This keyboard can have up to ten octaves, and can also sustain chords (built up by holding down "Shift" while clicking on the desired notes with a mouse), and then transpose these chords by a wide selection of intervals (combinations of arrow, shift and control keys). The keyboard can also be set so that certain notes are on and others are off. This is a tool for exploring modes. A large selection of modes (subsets) for any N element scale are included, and the user can specify others. Example 13 shows the nineteen note scale in a particular mode. This six note mode is "Yasser's Hexad", a basic harmonic unit described by Joseph Yasser in *A Theory of Evolving Tonality* (Yasser, 1932), and is made of intervals of 3 3 3 3 3 4 scale steps. The darker notes are non-modal notes, and don't play. The notes with dots on them are notes I selected to play a chord—a particular voicing of this hexad. I have read various criticisms of Yasser's choice of this hexad as the harmonic basis for his vision of nineteen tone harmony. On setting this mode up, hearing this chord, and playing with it in various trans–positions, it sounded fine to me. It sounded, in fact, like it could be used as the basis for a neo-impressionist kind of harmony. Since Yasser



EXAMPLE 12



EXAMPLE 13

had been trying to extend some of Scriabin's ideas, this was not surprising.

Scala also allows you to play chords and select them from a pre-set list. The software provides a selection of several hundred chords, and the list can be extended by the user. For any given scale, Scala will try to find the closest approximation for the chord in question. This could be used, for example, to hear the closest equivalent to a major (or any other) chord in higher order equal temperaments. For this piece, I made a vocabulary of chords based on selections of lower prime and Fibonacci numbers, a set of thirteen chords made with Andrew Culver and John Cage's IC program (Culver, 1993), and a set of chords which were stacks of lower just intervals. I placed these chords at the top of the list so that they were the first chords available when the chord window was opened.

Given the number of scales in the piece, and the number of pre-set chords I had available, even when I selected a particular chord, and

applied it to a desired scale, the number of resources was so vast that most often I would only have the most general idea of what the sonic result would be. This meant that even though I was performing directly, by selecting chords and playing them, there was still a high degree of improvisation and unpredictability in what I was doing. That is, the sheer number of resources often meant that this was still a process-oriented piece.

The piece was played on three laptop computers. Scala was running on each computer. On one of the computers, it was controlling a software synthesizer (Vaz Modular). On the other two, it produced MIDI output which controlled hardware synthesizers (Emu Proteus I and Korg XD5R). A family of similar timbres was programmed into each of these synthesizers. These were steady state tones (no vibrato) with a few upper harmonics, short attacks and decays of between one and two seconds. These timbres, while richer than sine waves, were still fairly plain, and the upper harmonics allowed the qualities of the harmonies to be heard a bit more easily than with sine waves. Often in performance, I would have one laptop playing an equal-temperament based scale, another playing the related Wilson-Finnamore scale, and the third playing my version of that scale. I would set up the exact same chord on the same root on all three computers, allowing the differences between the scales to create beats, richly dissonant harmonies, and a sense of the spatial shaping of the sound through an interaction of harmony and room acoustics. What I found especially remarkable was the way that the different chords combined to make a wide variety of complex timbres. Every time I introduced a new chord to the mix, the composite timbre changed dramatically. In fact, I could easily conceive of this piece being analysed primarily in terms of timbre, with all the scales and chords being considered as components of a unique kind of additive synthesis. My performance technique, then, involved me selecting scales, modes, chords, and timbres in real time, playing keyboard chords and melodies with the mouse, moving from laptop to laptop in order to do so, and gradually moving from family to family of these scales in an open-ended, improvisatory manner. Given the number of scales, modes, chords, and timbres I had available, it should be obvious that it will take a very very long time before the harmonic implications of this world are even revealed, much less exhausted. Performances of this piece have generally lasted in excess of two hours.

III: *Saturday in the Triakontahedron with Leonhard*

The third piece in the series is called *Saturday in the Triakontahedron with Leonhard*, a paraphrase of the Sondheim title, *Sunday in the Park with George*, and in it, I explore the harmonic resources of one part of another Wilson construction. In his paper "Euler Genus (3 5 7 9 11 13) Latticed within the Triakontahedron" (Wilson, 2001), he shows the scalar resources of the Euler-Fokker Genus made with factors 3 5 7 9 11 and 13, and some ways of dividing this scale into musically useful partitions. In my piece, I take one of these methods ("the cube of cubes"), and by exploring all possibilities of it, develop a series of 160 eight note scales, which I make available with an algorithmic perform–ance program developed using John Dunn's "Art Wonk" control program and Martin Fay's "Vaz Modular" software synthesizer.

The Euler-Fokker Genera are a whole family of scales made by making a matrix of Pythagorean scales. For better or worse, I've become fascinated by them. They are named for Leonhard Euler (from the eighteenth century) and Adriaan Fokker (from the twentieth). Adriaan Fokker developed certain scales based on multiplying harmonics 3, 5, and 7, and developed a series of microtonal organs to play approximations of them in 31 tone equal temperament, culminating with his 31 keyed organ of 1950. Fokker's "Genera of the third degree" is explained in Op de Coul 2000. Adriaan Fokker writes about the genesis of his microtonal organ and its unique keyboard in Fokker 1955. Briefly, a Euler-Fokker genus is any scale made by multiplying two or more intervals along axes to make a matrix. Some of the simplest Euler Fokker genera are simply Pythagorean scales. For example 10 genus of 3 3 3 would simply be a chain of three perfect 3/2 fifths (see Example 14).

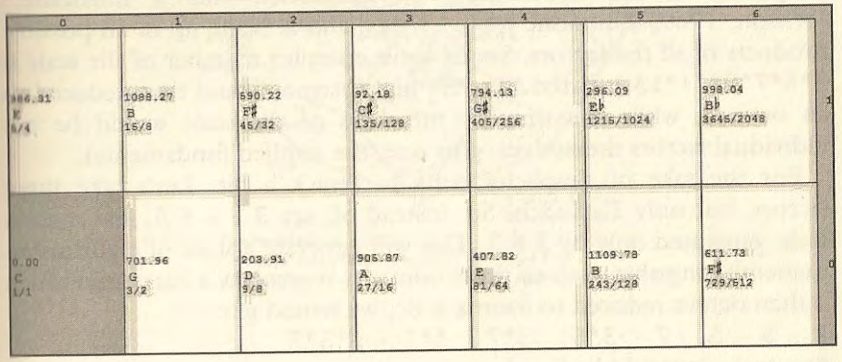
However, when two factors become involved, two and three dimen–sional matrices are generated. For example, a genus of 3 3 3 3 3 3 5 would be two chains of six 3/2 perfect fifths separated by a 5/4 major third, making a fourteen note scale. Example 15 (made with the "2D Play" option in Scala—this diagram is a mouse-performable keyboard) shows the structure of this scale.

This is a matrix of seven times two notes, a scale of fourteen notes. In terms of normal tuning, there are eight pitches represented once, with the E, B and F♯ each having two different versions. Notice that there is no F natural.

If you used three chains of intervals, you would then get a three dimensional lattice. Many people have used harmonic lattices like this, among them, Ben Johnston, James Tenney, and Larry Polansky. In Heidi von Gunden's *The Music of Ben Johnston* (Von Gunden, 1986,

C	(3/2)	G	(3/2)	D	(3/2)	A
0c	(702c)	702c	(702c)	1404c	(702c)	2106c
				204c		906c

EXAMPLE 14



EXAMPLE 15

p. 129), there is a good diagram of one lattice Ben Johnston uses, with 3/2 perfect fifths going vertically, while 5/4 major thirds go left to right, and 7/4 minor sevenths go front to back.

If you want to use more than three factors, then you'll get four, five, six, and n dimensional lattices. In fact, you'll get to the point where describing these formations as "lattices" will become largely futile. Things can quickly become very complex, with unmanageably large sets of pitches quickly being generated. If you want to have a scale made of, say, six factors, with each factor used just once, such as the Euler Genus (3 5 7 9 11 13), which would have all possible multipli–cations of all the factors, a lattice would be a totally inappropriate way to diagram the scale. More multidimensional ways of drawing the scale would have to be used. When Wilson tried to represent some of these four and higher dimensional scales graphically, (and even some lower dimensional scales with interesting properties), he realized that lattice notation would no longer serve. He described this as "breaking through the lattice barrier." (Wilson, personal communication).

Page one of Wilson's "Euler Genus (3 5 7 9 11 13)" paper consists of a diagram (Example 16), which is the most efficient way he has evolved to illustrate the structure of this scale. As can be seen from this diagram, it takes a very complex drawing to show all the "common factor" relationships in a scale like this. This scale is also a sort of larger "master scale" incorporating many of the "combination product set" scales—the hexanies, dekanies, eikosanies, etc. that Wilson had been proposing in his 1969 paper "D'alessandro—like a hurricane." (Wilson, 1969) This scale has 64 notes, and is made up of all possible products of all the factors. So the most complex member of the scale is 3*5*7*9*11*13 (treated as a very high harmonic and then reduced to an octave), while the simplest members of the scale would be the individual factors themselves, plus one, the implied fundamental.

For the sake of simplicity, let's backtrack a bit. Let's take three factors, but only 1 of each. So, instead of, say 3 3 3 5 5, lets make a scale generated only by 3 5 7. This will generate a scale of eight notes. Remembering that each of these numbers represents a harmonic which is then octave reduced to form a scale, we would get:

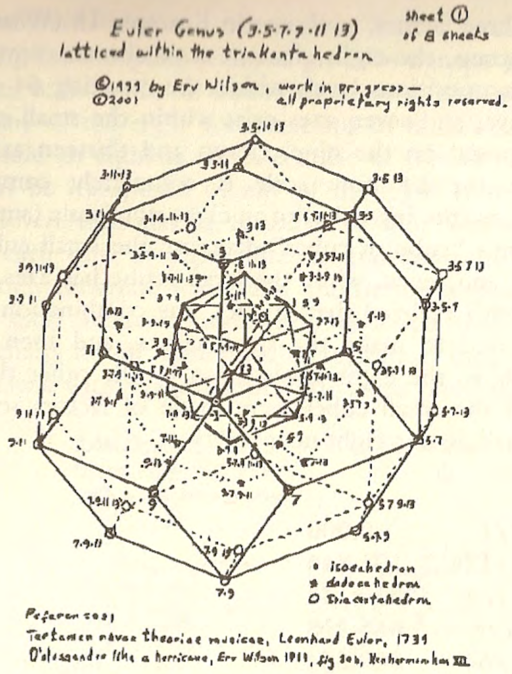
1 3 5 7 3*5 3*7 5*7 3*5*7

As a scale, it would be listed

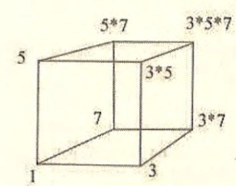
0:	1/1	0.000
1:	35/32	155.140
2:	5/4	386.314
3:	21/16	470.781
4:	3/2	701.955
5:	105/64	857.095
9:	7/4	968.826
7:	15/8	1088.269
8:	2/1	1200.000

And, if we wanted a graphic representation, since there are only three factors, we could notate it as Ervin Wilson does, as a cube, with each axis being a different generating interval, as shown in Example 17.

Any three factors can be combined to make a cubic eight element scale like this. Wilson shows that because there are six factors, the scale can be broken down into eight cubes like this, made of three factors, and these cubes can be placed at the corners of a larger cube, made of



EXAMPLE 16



EXAMPLE 17

the other three factors, as shown in Example 18 (Wilson, 2001, p. 4). In this diagram, the eight element scale shown above exists at eight different transpositions levels within the resulting 64 note scale. That is, three, five, and seven axes exist within the small cubes. These are then transposed on the nine, eleven and thirteen axes of the larger square. So the 64 note scale is completely comprised of eight transpositions (the big cube) of an eight note scale (small cubes).

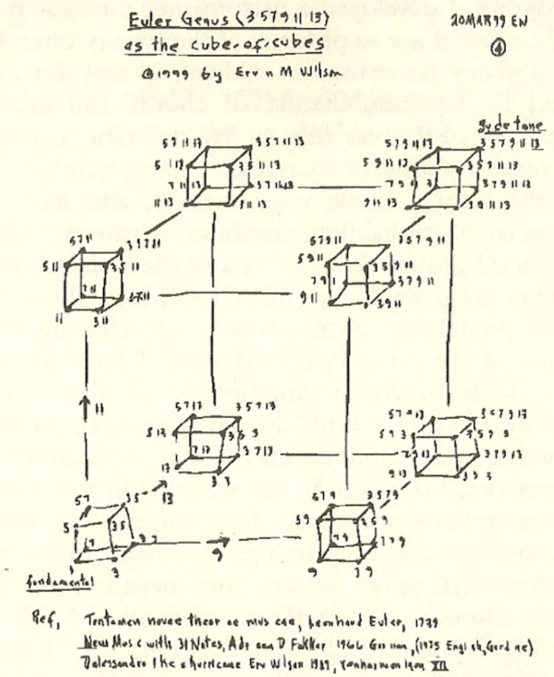
In Wilson's "cube of cubes" diagram, the small cubes have axes of three, five, and seven, while the larger cube has axes of nine, eleven, and thirteen. I realized that, in fact, any combination of three factors could be used to make the small cubes, and then they would be transposable to the eight levels given by the other three factors. For example, if the small cubes were made of factors seven, eleven and thirteen, making this eight note scale:

0:	1/1	0.000
1:	143/128	191.846
2:	77/64	320.144
3:	11/8	551.318
4:	91/64	609.354
5:	13/8	840.528
6:	7/4	968.826
7:	1001/512	1160.672
8:	2/1	1200.000

This scale could then be transposed eight ways, along the three, five and nine axes. In fact, there are twenty ways of combining sets of three out of six factors. Here they are:

3 5 7	9 11 13
3 5 9	7 11 13
3 5 11	7 9 13
3 5 13	7 9 11
3 7 9	5 11 13
3 7 11	5 9 13
3 7 13	5 9 11
3 9 11	5 7 13
3 9 13	5 7 11
3 11 13	5 7 9

And each of these three factor combinations can be combined with any of the combinations of the other three factors (out of the six factors generating the complete scale) to produce transpositions . That is, an eight note scale made of factors three, five, and seven can be transposed by any of the eight combinations of factors nine, eleven, and thirteen, while an eight note scale made of factors nine, eleven, and thirteen can be transposed by any of the eight combinations of three, five, and seven, thus making actually a total of twenty eight-note scales that can each exist at eight different transposition levels. This seems like enough different scales and transposition possibilities to keep even the most strident modulation obsessed tonal composer happy. I made charts for all these possible scales and their reciprocals. They make a matrix with vertical scales providing transposition levels



EXAMPLE 18

for the horizontal scales, and vice-versa. The first two of these matrices are shown here in Examples 19 and 20.

In Example 19, rows read from left to right are scales made of factors of three, five, and seven, columns read vertically are scales made of factors nine, eleven, and thirteen. The top number of each set of three is the ratio of the scale degree. The next number is the number of cents the tone is above the fundamental. The bottom number is the scale degree of the tone. The numbers in parentheses are scale degrees in a second (upper) octave. The tuning files for Vaz Modular allow any 128 tones to be assigned a microtonal retuning. So two octaves of the scale will fit in the tuning file. This lessens the necessity for changing octaves in the oscillators as often as might otherwise be the case, and also allows transpositions and modes to be played more freely.[4]

Using ArtWonk, I developed a performance interface for this piece. This interface allowed me to pick any of the twenty three-factor eight-note scales, and any transposition of them, in real time, and to hear them applied to any combination of chords and melodies, using frequency modulated timbres (fm) in five different registers. Three of these lines, one consisting of four-voice chords, another consisting of dyads, and the third a single voice melody, also had their pitches selected by a set of probability distributions (sieves), which allowed only certain chords, intervals, or subsets of the scale to be played. The other two lines were single-line melodies, which chose their pitches using similar probability distributions, but selecting from all the available tones of the scale. Pan, volume, and note duration are also controlled from this interface. Additionally, another control allowed selection of overall tempo. With this interface, I could freely explore any of the scales, and a large number of harmonic subsets in each of them. Furthermore, I could make this selection in real time, being able to perform changes and hear their effects immediately. What was most delightful to me was that even though the melodies and chords were randomly selected (through sieves), thus negating any intentionally composed directionality within them, when one changed from one scale to another, it sounded like a modulation, often a quite dramatic one. On listening back to a recording of a performance of the piece, I was quite amazed that for me, both local and long term harmonic motion seemed to be happening. And furthermore, this sense of harmonic motion was also occurring while moving between harmonic worlds that were relatively consonant (the 3 5 7 factor scale) or relatively dissonant (the 9 11 13 scale). That is, harmonic motion was heard as both a function of changing probability distributions within a scale, and also as a consequence of moving between scales. And the

1/1	35/32	5/4	21/13	3/2	105/64	7/4	15/8
0	155.1	386.3	470.8	702.0	85701	968.8	1088.3
0	8	21	25	38	45	52	58
143/128	5005/4096	715/512	3003/2048	429/256	15015/8192	1001/512	2145/2048
191.8	347.0	578.2	662.6	893.9	1045.9	1160.8	80.114
10	19	31	35	47	56	61	5 (69)
9/8	315/256	45/32	189/128	27/16	945/512	63/62	135/128
203.9	359.0	590.2	674.7	905.9	1061.1	1172.7	92.2
11	20	32	36	48	57	62	6 (70)
1287/1024	45095/32768	6435/4096	27027/16384	3861/2048	135135/131072	9009/8192	19305/16384
395.8	550.9	782.1	866.5	1097.7	52.9	164.6	284.0
22	29	42	46	59	3 (67)	9 (73)	15 (79)
11/8	385/256	55/32	231/128	33/32	1155/1024	77/64	165/128
551.3	706.5	937.6	1022.1	53.3	208.4	320.1	439.6
30	39	51	54	4 (68)	12 (76)	17 (81)	24 (88)
99/64	3465/2048	495/256	2079/2048	297/256	10395/8192	693/572	1485/1024
755.2	910.4	1141.5	26.0	257.2	412.3	524.1	643.5
41	49	60	1 (65)	14 (78)	23 (87)	27 (91)	34 (98)
13/8	455/256	65/64	273/256	39/32	1365/1024	91/64	195/128
840.5	995.7	26.8	111.3	342.5	497.6	609.4	728.8
44	53	2 (66)	7 (71)	18 (82)	26 (90)	33 (97)	40 (104)
117/64	4095/2048	585/512	2457/2048	351/256	12285/8192	819/512	1755/1024
1044.4	1199.6	230.7	315.2	546.4	701.6	813.3	932.7
55	63	13 (77)	16 (80)	28 (92)	37 (101)	43 (107)	50 (114)

EXAMPLE 19: EULER GENUS 3 5 7 VS. 9 11 13 AS EIGHT PARALLEL 8 NOTE SCALES WITH SCALE DEGREE 0–63 FOR VAZ. 1: 3–5 × 9–11–13.

1/1	135/138	9/8	5/4	45/32	3/2	27/16	15/8
0	92.2	203.9	386.3	590.2	702.0	905.9	1088.3
0	6	11	21	32	38	48	58
143/128	19305/16384	1287/1024	715.512	6435/4096	429/256	3861/2048	2145/2048
191.8	284.0	395.8	578.2	782.1	893.8	1097.7	80.1
10	15	22	31	42	47	59	5 (69)
77/64	10395/8192	693/512	385/256	3465/2048	231/128	2079/2048	1155/1024
320.1	412.3	524.0	706.4	910.4	1022.1	26.0	208.4
17	23	27	39	49	54	1 (65)	12 (76)
11/8	1485/1024	99/64	55/32	495/2569	33/32	297/256	165/128
551.3	643.5	755.2	937.6	1141.5	53.3	257.2	439.6
30	34	41	51	60	4 (68)	13 (78)	24 (88)
91/64	12285/8192	819/512	455/256	4095/2048	273/256	2457/2048	1365/1024
609.4	701.5	813.3	995.7	1199.6	111.3	315.2	497.6
33	37	43	53	63	7 (71)	16 (80)	26 (90)
13/8	1755/1024	117/64	65/64	585/512	39/32	351/256	195/128
840.5	932.7	1044.4	26.8	230.7	342.5	546.4	728.8
44	50	55	2 (66)	13 (77)	18 (82)	28 (92)	40 (104)
7/4	945/512	63/32	35/32	315/256	21/16	189/128	105/64
968.8	1061.0	1172.7	155.1	359.1	470.8	674.7	857.1
52	57	62	8 (72)	20 (84(	25 (89)	36 (100)	45 (109)
1001/512	135135/131072	9009/8192	5005/4096	45045/32768	3003/2048	27027/16384	15015/8192
1160.7	52.9	164.6	347.0	550.9	662.6	866.59	1048.9
61	3 (67)	9 (73)	19 (83)	29 (91)	35 (99)	46 (110)	56 (120)

EXAMPLE 20: EULER GENUS 3 5 9 VS. 7 11 13 AS EIGHT PARALLEL 8 NOTE SCALES WITH SCALE DEGREE 0–63 FOR VAZ. 2: 3–5 VS 7–11–13.

overall "feel" of the harmony—whether more consonant or more dissonant—was clearly changing as well. This is unlike, say, in tonal harmony in the twelve note scale, where the overall harmonic "feel" doesn't change, even though scale type and fundamental may. That is, one might change from major to minor scales, but intervals used remain the same. Here, even the kinds of basic intervals one is using change when one changes scales. In short, with this 64 note scale, partitioned in this "cube of cubes" way, I'd arrived at what I felt was a true sense of expanded tonality, one that also allowed an expanded kind of melodic vocabulary (non-directional melodies, as well as melodies with functional harmonic structuring) to exist within it. The fm timbres chosen were also a bit more harsh and spiky than the timbres used in the first two pieces, but not to such an extent that pitch perception was affected. One can still clearly hear the fundamentals of these pitches, and how they combine harmonically. We're still dealing with (mostly) harmonic-series timbres. This piece is one that I feel I haven't performed enough yet. Many more hours of exploration are needed, I feel, to hear a wide range of the harmonic and melodic possibilities of this complex. I eagerly await further performance opportunities for this piece.

Working with these three large mathematical-musical worlds, I keep getting hints that all three relate in some way. To borrow a conceit from physics, I think there may be some sort of "unified-harmonic-field" theory at work here, which would unite all three methods of scale formation (additive sequences, the scale tree, the Euler-Fokker genera) under one sort of mathematical procedure. Unfortunately, these have so far been only hints, illusive intuitions that there might be some kind of deeper structure at work here. In "The Triangle/Lambda Equivalence" (Wilson, 2003), Wilson proves that the Pingala/Pascal triangle, and the Farey Series/Lambdoma diagrams are equivalent. And Brian McLaren, in "General Methods for Generating Musical Scales" (McLaren, 1991) briefly discusses the underlying relationship of scales made with additive series or processes of all kinds. But much more work needs to be done here. I hope that some of the more mathematically minded readers of this article might work further on this, in order to discover if there is some larger underlying principle behind the structures of all these scales.

Notes

1. Nor are these the only scales derivable in this manner. In *Some Exploratory Triangles for Scales of Mt. Meru* (Wilson 2002), Wilson shows other triangles. For example, instead of a triangle with ones on either descending edge, he proposes a triangle with fours down the left edge and sevens down the right edge. He gives examples of seventeen different triangles, in fact, each with a different set of generating "edge numbers." Each of these seventeen triangles, with diagonals similar to Example 1, generates one, and sometimes two different additive series, all of which use the same formula as A, and have the same convergence interval of Phi. Preliminary investigations lead me to believe that diagonals similar to Example 2 will generate additive series with the same formula and convergence interval of series B, and diagonals similar to series C will generate additive series with the same formula and convergence interval of series C, etc.
2. There are, of course, much more elaborate ways of relating tuning and timbre. The classic study of this is *Tuning Timbre Spectrum Scale* by William Sethares (Sethares, 2005).
3. To see the diagram in all its full-color glory, the reader is referred to http://www.elvenminstrel.com/music/tuning/horagrams/davidstree.htm.
4. There are, of course, other ways of partitioning this scale. In *Memories of Cecil Street on a Hot Summer Day* from *Playing in Traffic* (2001) I used a partition of the scale into four mutually exclusive sixteen note subsets. There are fifteen unique ways to do this, resulting in a family of 60 related scales. And there are many other ways of working with the Euler-Fokker genera. For example, scale size might be a determining factor. If one wanted, say, to work with scales of only twelve notes, using seven generating factors, this would give rise to a family of 196 related twelve-note scales. Using six generating factors would yield 126 related twelve-note scales, etc.)

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